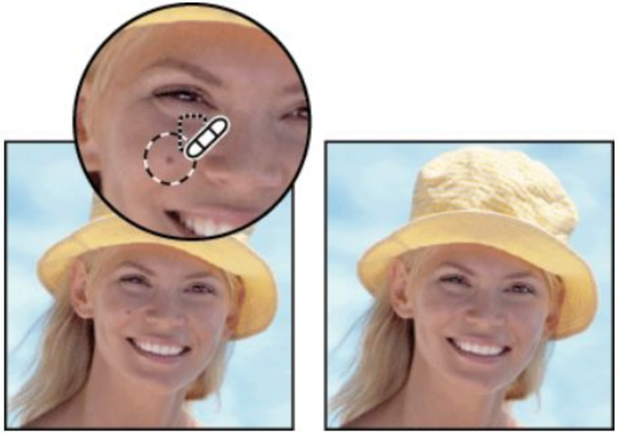
The Differential Geometry of the Healing Brush

The Healing Brush

- Using samples from nearby pixels, the healing brush corrects imperfections
- The healing brush only takes a few seconds to work
- The healing brush works by solving a 4th order partial differential equation!



Using the Spot Healing Brush to remove a blemish

Photoshop Healing Brush: A Tool for Seamless Cloning (Applications of CV Workshop @ ECCV 2004)

- A nice and short explanation of how the healing brush works.
- The idea is to approximate the image using a *biharmonic function*, then reconstruct the target area with the biharmonic approximation.

Photoshop Healing Brush: a Tool for Seamless Cloning

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Abstract. The Healing Brush is a tool first introduced in Photoshop, that achieves seamless removal of defects in images. A similar image processing algorithm, called Poisson Editing, was later proposed in [8]. Our paper presents the theoretical ideas on which Healing is based, as well as some implementational details. Healing is performed by constructing iterative solutions to a fourth order partial differential equation. Its solutions follow the spatial variations between pixels in a sampled area, while at the same time are continuous and have continuous derivatives at the boundary. Poisson Editing is only continuous at the boundary. Also, our exact equation describes cloning of features from one image to another with greater fidelity. Our mathematical understanding of the process is based on viewing the image as a section in a fibred manifold, and minimizing certain natural expression for the energy written in terms of connections. In this we are, in a way, following the line of thought in Gauge Theories in Physics.

What is a biharmonic function?

- Let $f : \mathbf{R}^2 \to \mathbf{R}$ be a function which maps a pixel to a color (single channel for clarity).
- A function is **harmonic** if $\nabla \cdot \nabla f = 0$, meaning the solution to the Laplace equation is 0.

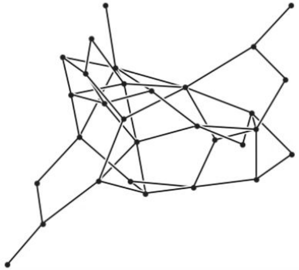
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \cdot \begin{bmatrix} D_1 f \\ D_2 f \\ D_3 f \end{bmatrix} = 0$$

- f is **biharmonic** if $\nabla^2 \nabla^2 f = 0$

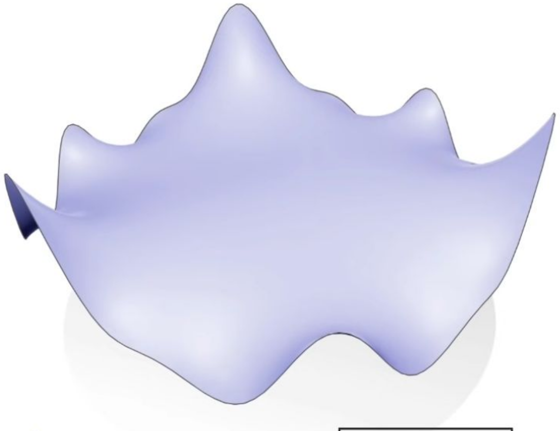
Intuition behind harmonic functions

Laplace equation

- Suppose we keep boundary values fixed, and run the heat equation for a *very* long time…
- Eventually, value at each point will *equal* the average value in a small neighborhood
- Resulting function is called "harmonic," solution to *Laplace equation*
- Graph analogy: everybody in a social network is, on average, just as intelligent as all their friends



$$\frac{d}{dt}u = \Delta u, \quad u|_{\partial\Omega} = g$$



$$\frac{d}{dt}u = 0 \quad \Longrightarrow \quad \boxed{\Delta u = 0}$$

Laplace equation

Solving the Laplace Equation (Heat Equation) on an Image

- For a *harmonic solution*, just convolve many times!

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- Only overwrite the user-selected area.

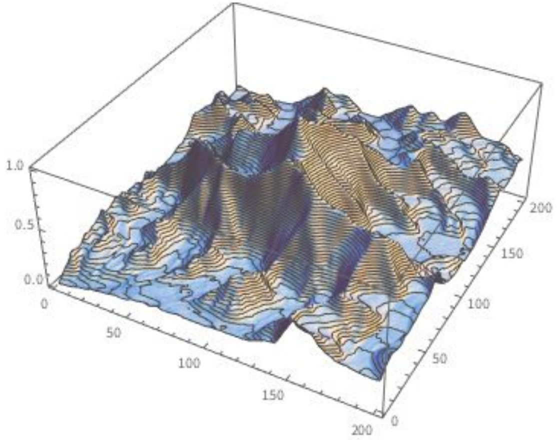
- For a *biharmonic solution,* use this!

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -2 & 8 & -2 & 0 \\ -1 & 8 & 12 & 8 & -1 \\ 0 & -2 & 8 & -2 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

- Unfortunately, the results of convolving are too smooth. We need something more complicated

Why convolution doesn't work well

- Suppose we represent an image as an elevation map: **R2 x R**
- **Observation:** Pixels with higher z value do not always appear brighter
  - The way we perceive brightness depends on a pixel's neighbors
  - Solving the heat equation by "averaging" neighbors makes images that are too smooth
- Instead of representing derivatives as relative to a pixel's neighbors, the healing brush represents changes in brightness as a ***covariant derivative (connection)***.



Representing images as a fiber bundle

- The *covariant derivative* assigns a different derivative “scale” to every pixel
- The argument for using a covariant derivative is that humans perceive colors differently depending on its neighbors
- A *fiber bundle* is a topological space that is locally a product space.